Biologically Inspired Distributed Estimation and Control

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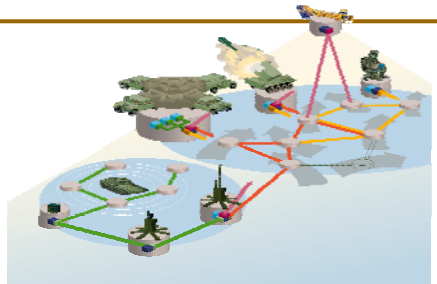
Method…

observation of natural networks → formal understanding of design principles → "explanation" of observed natural behavior / design of algorithms for artificial networks

Large-scale networks of multi-agent system:
limitations imposed by network structure

in collaboration with P. Barooah (UCSB), A. Swami (ARL)

Goals & impact



multitude of sensors feeding real-time information to soldiers, command & control centers, and autonomous systems

Project objectives:

1. Determine fundamental asymptotic scalability limits for computation and control in large-scale, unreliable networks
2. Design distributed algorithms that operate at or just below the theoretical limit

Impact for the army:

Deliver algorithms and architectures for low-cost, reliable, high-performance large-scale networked systems.

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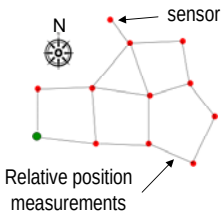
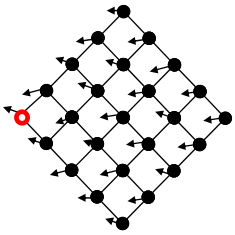
Prototypical multi-agent problems

we seek for foundational results applicable to large classes of problems, but that have near-term impact in army-relevant problem

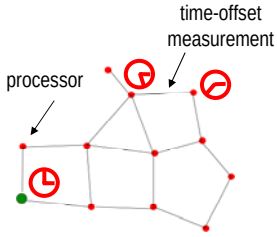
Localization



Formation control



Time synchronization



1. Design of distributed algorithms
   (low communication, low computation, robust w.r.t. agent/communication faults)
2. Impact of network structure on system performance
   (scalability w.r.t. number of nodes/edges)

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Multi-agent coordination in nature

v-shape reduces drag

synchronized hunting

confuses predators

Features common to problems in both natural and artificial domains:

- Motion coordination benefits team of agents
- The agents' objective determines the shape for the formation
- For scalability, each agent must compute its control based on local sensing

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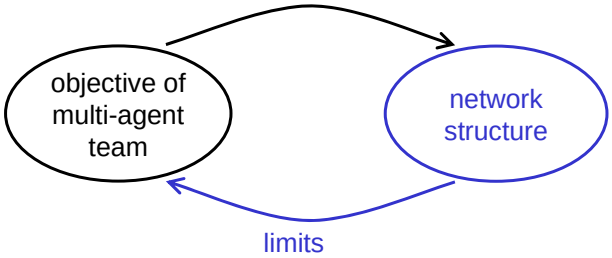
Distributed control/estimation

v-shape reduces drag

synchronized hunting

confuses predators



Consequence of…
"each agent must compute its control based on local sensing"

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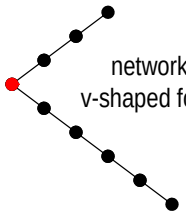
V-shapes

Intrinsic limitations imposed by the *structures of the network* on the agent's coordination…



network for a v-shaped formation

Suppose agent wants to determine its position with respect to the leader

- solely based on local measurements

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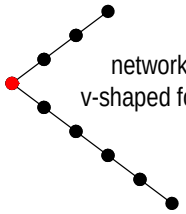
V-shapes

Intrinsic limitations imposed by the *structures of the network* on the agent's coordination…



network for a v-shaped formation

Suppose agent wants to determine its position with respect to the leader

- solely based on local measurements

For a v-shaped network:

$$\underbrace{\text{error variance} \approx \text{\# of edges to leader} \times \text{error variance of each edge}}$$

errors grow linearly with # of agents $\Rightarrow$ acceptable variance limits # of agents $\Rightarrow$ not a scalable sensing network

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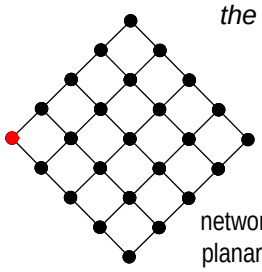
Planar lattices

Intrinsic limitations imposed by the *structures of the network* on the agent's coordination…



network for a planar lattice

Suppose agent wants to determine its position with respect to the leader

- solely based on local measurements

For a planar lattice network:

error variance $\propto$ Log(# of edges to leader) $\times$ error variance of each edge

errors grow with log of # of agents $\Rightarrow$ multiplying # agents by 10 only increases error by an additive factor $\Rightarrow$ much better scaling properties

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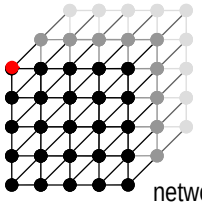
3-Dimensional lattices

Intrinsic limitations imposed by the *structures of the network* on the agent's coordination…



network for a 3-d lattice

Suppose agent wants to determine its position with respect to the leader

- solely based on local measurements

For a 3-d lattice network:

error variance $\propto$ error variance of each edge

errors grow remains bounded even as # agents $\rightarrow \infty$ $\Rightarrow$ truly scalable sensing network

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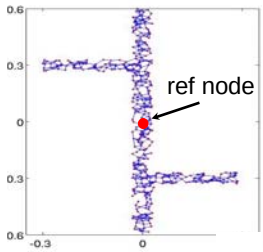
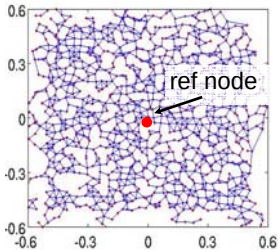
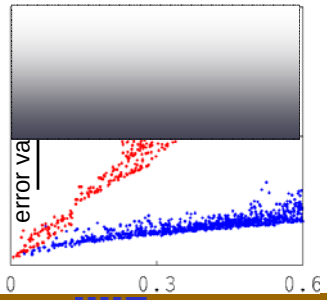
What makes general graphs scalable?



same total
# of nodes,
same total
# of measurements
(edges)





Which is which?

red dots → left network

blue dots → right network

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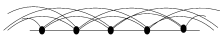
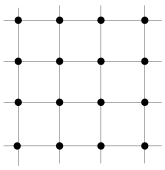
What makes general graphs scalable?

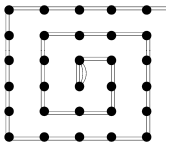
degree 2
linear growth

degree 6
linear growth

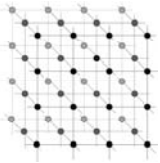




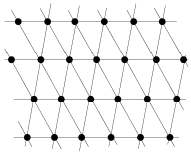
degree 4
log growth



degree 4
linear growth



degree 6
no growth



degree 6
log growth

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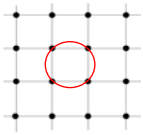
Dimension of a graph

A graph is said to be dense in d-dimensions if
it can be drawn in a d-dimensional space so that:

1. One cannot fit arbitrarily large balls between nodes

$d = 2$

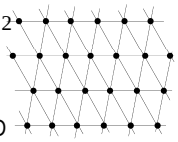


node density

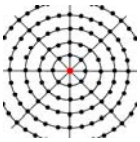
2. The graphical distance between nodes grows at most linearly with the geometric distance between nodes

edge density

$d = 2$

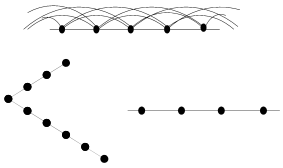


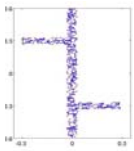
dense in 2D
(small geometrical distance means small graphical distance)



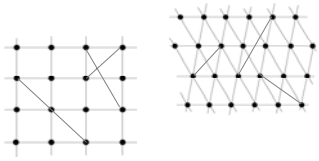
not dense in 2D
(small geometrical distance does not mean small graphical distance)

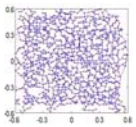
Error propagation vs. dimension



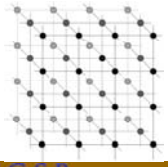


For graphs dense in 1D
error variance $\leq$ O(# edges)





For graphs dense in 2D
error variance $\leq$ O(Log # edges)



For graphs dense in 3D
error variance $\leq$ constant

OSE distributed algorithm

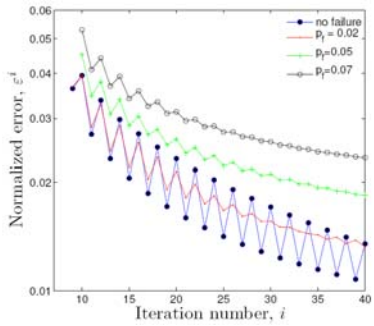
So far … intrinsic limitations imposed by the network structure.
Can we find algorithms that achieve this?

Suppose agents wants to determine position with respect to the leader

- solely based on local measurement
- with limited & unreliable communication
- with limitation computational power

Overlapping Subgraph Estimator (OSE) algorithm

1. iterative algorithm
2. at each step, every node solves a small localization problem and broadcasts its solution to its immediate neighbors
3. algorithm converges to optimal estimate
4. algorithm is robust to communication failures



Motion control algorithm

$$\dot{x}_u = -c_u \sum_{v \text{ neighbor of } u} \Big(\overbrace{x_u - x_v}^{\text{position of u relative to v}} - \underbrace{r_{u,v}}_{\text{desired position of u relative to v}} \Big)$$

agent u's velocity

directs agent so as to decreases error with respect to immediate neighbors

each agent only needs to measure its position with respect to nearby neighbors
⇓
distributed control

(closely related to Craig Reynolds' model used in computer graphics to produce natural flocking and schooling behavior)

Scalability

Robustness with respect to measurement noise…

$$\dot{x}_u = -c_u \sum_{v \text{ neighbor of } u} \left(x_u - x_v - r_{u,v} + n_{u,v} \right)$$

For graphs dense in 1D:
tracking error variance $\leq$ O(# edges)

For graphs dense in 2D:
tracking error variance $\leq$ O(Log # edges)

For graphs dense in 3D:
tracking error variance $\leq$ constant

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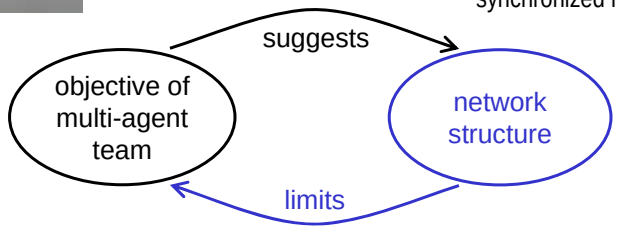
Conclusion

v-shape reduces drag

synchronized hunting



confuses predators

- *Network structure imposes fundamental limitation on control/estimation performance*
- *There are distributed algorithms that operate just below the theoretical limits*

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Summary

observation of natural networks → formal understanding of design principles → "explanation" of observed natural behavior / design of algorithms for artificial networks

Large-scale networks of multi-agent system:
limitations imposed by network structure

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